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# On the Atmospheric Neutrino Oscillations, $\theta_{13}$ and Neutrino Mass Hierarchy*

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**Abstract**

We give predictions for the up-down asymmetry in the Nadir angle dependence of the ratio $N_\mu/N_e$ of the rates of the $\mu-$like and $e-$like multi-GeV events measured in water-Čerenkov detectors (Super-Kamiokande, etc.) in the case of 3-neutrino oscillations of the atmospheric $\nu_e$ ($\bar{\nu}_e$) and $\nu_\mu$ ($\bar{\nu}_\mu$), driven by one neutrino mass squared difference, $|\Delta m^2_{\rm atm}| \equiv |\Delta m^2_{31}| \sim (2.0 - 3.0) \times 10^{-3}$ eV$^2 \gg \Delta m^2_{21} \equiv \Delta m^2_\odot$. This ratio is particularly sensitive to the Earth matter effects in the atmospheric neutrino oscillations, and thus to the values of $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$, $\theta_{13}$ and $\theta_{23}$ being the neutrino mixing angle limited by the CHOOZ and Palo Verde experiments and that responsible for the dominant atmospheric $\nu_\mu \to \nu_\tau$ ($\bar{\nu}_\mu \to \bar{\nu}_\tau$) oscillations, respectively. It is also sensitive to the type of neutrino mass spectrum which can be with normal ($\Delta m^2_{\rm atm} > 0$) or with inverted ($\Delta m^2_{\rm atm} < 0$) hierarchy.

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# 1 Introduction

There has been a remarkable progress in the studies of neutrino oscillations in the last several years. The experiments with solar, atmospheric and reactor neutrinos [1, 2, 3, 4, 5] have provided compelling evidences for the existence of neutrino oscillations driven by nonzero neutrino masses and neutrino mixing. Evidences for oscillations of neutrinos were obtained also in the first long baseline accelerator neutrino experiment K2K [6].

The latest addition to this magnificent effort is the evidence presented at this Workshop by the Super-Kamiokande (SK) collaboration for an "oscillation dip" in the $L/E-$dependence, of the (essentially multi-GeV) $\mu-$like atmospheric neutrino events [7], $L$ and $E$ being the distance traveled by neutrinos and the neutrino energy. This beautiful result represents the first ever observation of a direct effect of the oscillatory dependence on $L/E$ of the probability of neutrino oscillations in vacuum.

An improved analysis of SK atmospheric neutrino data, performed recently by the SK collaboration, gave [8] at 90% C.L.

$$1.3\,\times\,10^{-3}\,\mathrm{eV}^2\;\leq\;|\Delta m^2_{\rm atm}\,|\;\leq\;3.1\,\times\,10^{-3}\,\mathrm{eV}^2\;, \qquad 0.90 \leq \sin^2 2\theta_{23} \leq 1.0\;, \tag{1}$$

with best fit values $|\Delta m^2_{\rm atm}\,| \equiv \Delta m^2_{31} = 2.0\times 10^{-3}$ eV$^2$ and $\sin^2 2\theta_{23} = 1.0$ (see also ref. [9]). Earlier analysis of the SK atmospheric neutrino data produced somewhat larger values of $|\Delta m^2_{\rm atm}\,|$: the best fit value found, e.g., by SK collaboration [8] reads $|\Delta m^2_{\rm atm}\,| = 2.5\times 10^{-3}$ eV . Finally, the values of $|\Delta m^2_{\rm atm}\,|$ and $\sin^2 2\theta_{23}$, deduced from the SK analysis of the $L/E$ dependence of the observed $\mu-$like atmospheric neutrino events [7], are compatible with the values obtained in the other analysis.

As is well-known, the atmospheric neutrino and K2K data do not allow one to determine the signs of $\Delta m^2_{\rm atm}$ and of $\cos 2\theta_{23}$ when $\sin^2 2\theta_{23} \neq 1.0$. The two possibilities, $\Delta m^2_{\rm atm} > 0$ and $\Delta m^2_{\rm atm} < 0$, correspond to two different types of neutrino mass spectrum: with normal hierarchy (NH), $m_1 < m_2 < m_3$, and with inverted hierarchy (IH), $m_3 < m_1 < m_2$. The ambiguity in the sign of $\cos 2\theta_{23}$ implies that when, e.g., $\sin^2 2\theta_{23} = 0.92$, two values of $\sin^2\theta_{23}$ are possible, $\sin^2\theta_{23} \cong 0.64$ or 0.36.

A very important parameter in the phenomenology of 3-neutrino mixing and oscillations is the angle $\theta_{13}$, limited by the data from the CHOOZ and Palo Verde experiments. The precise limit on $\theta_{13}$ is $\Delta m^2_{\rm atm}$ – dependent (see, e.g, ref. [10]). Using the 99.73% allowed range of $\Delta m^2_{\rm atm} = (1.1-3.2)\times 10^{-3}$ eV$^2$ from ref. [9], one gets from a combined 3-neutrino oscillation analysis of the solar neutrino, CHOOZ and KamLAND data [11]:

$$\sin^2\theta_{13} < 0.047\ (0.074), \qquad 90\%\ (99.73\%)\ \text{C.L.} \tag{2}$$

The global analysis of the solar, atmospheric and reactor neutrino data performed in ref. [12] gives $\sin^2\theta_{13} < 0.054$ at 99.73% C.L.

Getting more precise information about the value of the mixing angle $\theta_{13}$, determining the sign of $\Delta m^2_{\rm atm}$ , or the type of the neutrino mass spectrum (with normal or inverted hierarchy), and measuring the value of $\sin^2\theta_{23}$ with a higher precision is of fundamental importance for the progress in the studies of neutrino mixing.

In ref. [13] we have derived predictions for the Nadir angle ($\theta_n$) dependence of the ratio $N_\mu/N_e$ of the rates of the $\mu-$like and $e-$like multi-GeV events measured in water-Čerenkov detectors in the case of 3-neutrino oscillations of the atmospheric $\nu_e$ ($\bar{\nu}_e$) and $\nu_\mu$ ($\bar{\nu}_\mu$), driven by one neutrino mass squared difference, $|\Delta m^2_{\rm atm}| \sim (2.0-3.0)\times 10^{-3}$ eV$^2 \gg \Delta m^2_\odot$. This ratio was shown to be particularly sensitive to the Earth matter effects in the sub-dominant $\nu_\mu \leftrightarrow \nu_e$ ($\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$) oscillations of the atmospheric $\nu_\mu$ ($\bar{\nu}_\mu$) and $\nu_e$ ($\bar{\nu}_e$) [15, 16], and thus i) to the value of $\sin^2\theta_{13}$ which drives the subdominant oscillations, ii) to the value of $\sin^2\theta_{23}$ which determines the maximal possible value of the corresponding subdominant transition probabilities, and iii) to the type of neutrino mass spectrum, i.e., the sign of $\Delta m^2_{\rm atm}$. It was shown in ref. [13], in particular, that for $\sin^2\theta_{13} \gtrsim 0.01$, $\sin^2\theta_{23} \gtrsim 0.5$ and at $\cos\theta_n \gtrsim 0.4$, the Earth matter effects modify substantially the $\theta_n-$ dependence of the ratio $N_\mu/N_e$ and in a way which cannot be reproduced with $\sin^2\theta_{13} = 0$ and a different value of $\sin^2\theta_{23}$. For normal hierarchy, the effects of interest can be as large as $\sim 25\%$ for $\cos\theta_n \sim (0.5-0.8)$, can reach $\sim 35\%$ in the Earth core bin $\cos\theta_n \sim (0.84-1.0)$, and might be observable [13]. They were shown to be typically by $\sim 10\%$ smaller in the inverted hierarchy case. This permitted to conclude that an observation of the Earth matter effects in the Nadir angle distribution of the ratio $N_\mu/N_e$ would clearly indicate that $\sin^2\theta_{13} \gtrsim 0.01$ and $\sin^2\theta_{23} \gtrsim 0.50$.

In the present article we give predictions for the up-down (U-D) asymmetry (see also ref. [14]) in the Nadir angle dependence of the ratio $N_\mu/N_e$ of the rates of the $\mu-$like and $e-$like multi-GeV events measured in water-Čerenkov detectors (Super-Kamiokande, etc.). As in ref. [13], we consider the case of 3-neutrino oscillations of the atmospheric $\nu_e$ ($\bar{\nu}_e$) and $\nu_\mu$ ($\bar{\nu}_\mu$), driven by one neutrino mass squared difference, $|\Delta m^2_{\rm atm}| \sim (2.0-3.0)\times 10^{-3}$ eV$^2 \gg \Delta m^2_\odot$. The indicated U-D asymmetry is expected to have substantially smaller systematic uncertainty than (the Nadir angle dependence of) the ratio $N_\mu/N_e$ itself.

# 2 Effects of Subdominant 3-$\nu$ Oscillations

The subdominant $\nu_\mu \to \nu_e$ ($\bar{\nu}_\mu \to \bar{\nu}_e$) and $\nu_e \to \nu_{\mu(\tau)}$ ($\bar{\nu}_e \to \bar{\nu}_{\mu(\tau)}$) oscillations of the multi-GeV atmospheric neutrinos of interest should exist and their effects could be observable if three-flavor-neutrino mixing takes place in vacuum, i.e., if $\sin^2 2\theta_{13} \neq 0$, and if $\sin^2 2\theta_{13}$ is sufficiently large [15, 16, 17, 18] (see also ref. [19]). These transitions are driven by $\Delta m^2_{\rm atm}$. The probabilities of these transitions contain $\sin^2\theta_{23}$ as factor which determines their maximal value. For $\Delta m^2_{\rm atm} > 0$, the $\nu_\mu \to \nu_e$ ($\bar{\nu}_\mu \to \bar{\nu}_e$) and $\nu_e \to \nu_{\mu(\tau)}$ ($\bar{\nu}_e \to \bar{\nu}_{\mu(\tau)}$) transitions of the multi-GeV atmospheric neutrinos (antineutrinos) are amplified (suppressed) by the Earth matter effects; if $\Delta m^2_{\rm atm} < 0$, the transitions of $\nu_\mu$, $\nu_e$ are suppressed and those of $\bar{\nu}_\mu$, $\bar{\nu}_e$ are enhanced. Therefore for a given sign of $\Delta m^2_{\rm atm}$, the Earth matter affects differently the transitions of neutrinos and antineutrinos. Thus, the study of the subdominant atmospheric neutrino oscillations can provide information, in particular, about the sign of $\Delta m^2_{\rm atm}$ and the magnitudes of $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$.

Under the condition $|\Delta m^2_{\rm atm}| \gg \Delta m^2_\odot$, the relevant three-neutrino $\nu_\mu \to \nu_e$ ($\bar{\nu}_\mu \to \bar{\nu}_e$) and $\nu_e \to \nu_{\mu(\tau)}$ ($\bar{\nu}_e \to \bar{\nu}_{\mu(\tau)}$) transition probabilities reduce effectively to a 2-neutrino

transition probability [20] with $\Delta m^2_{\rm atm}$ and $\theta_{13}$ playing the role of the relevant two-neutrino oscillation parameters.

The fluxes of atmospheric $\nu_{e,\mu}$ of energy $E$, which reach the detector after crossing the Earth along a given trajectory specified by the value of $\theta_n$, $\Phi_{\nu_{e,\mu}}(E,\theta_n)$, are given by the following expressions in the case of the three-neutrino oscillations under discussion [16]:

$$\Phi_{\nu_e}(E,\theta_n) \cong \Phi^0_{\nu_e}\ \Big(1 + [s^2_{23} r - 1]\ P_{2\nu}\Big)\,, \tag{3}$$

$$\Phi_{\nu_\mu}(E,\theta_n) \cong \Phi^0_{\nu_\mu}\ \Big(1 + s^4_{23}\ [(s^2_{23}\ r)^{-1} - 1]\ P_{2\nu}$$
$$-2c^2_{23}s^2_{23}\ \Big[1 - Re\ (e^{-i\kappa} A_{2\nu}(\nu_\tau \rightarrow \nu_\tau))\Big]\Big)\,. \tag{4}$$

Here $\Phi^0_{\nu_{e(\mu)}} = \Phi^0_{\nu_{e(\mu)}}(E,\theta_n)$ is the $\nu_{e(\mu)}$ flux in the absence of neutrino oscillations and

$$r \equiv r(E,\theta_n) \equiv \frac{\Phi^0_{\nu_\mu}(E,\theta_z)}{\Phi^0_{\nu_e}(E,\theta_z)}\ , \tag{5}$$

$P_{2\nu} \equiv P_{2\nu}(\Delta m^2_{\rm atm}\ ,\theta_{13};E,\theta_n)$ is the probability of two-neutrino $\nu_e \rightarrow \nu'_\tau$ oscillations in the Earth, where $\nu'_\tau = s_{23}\nu_\mu + c_{23}\nu_\tau$, and $\kappa$ and $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau)$ are known phase and 2-neutrino transition probability amplitude [20, 15, 16, 13].

For the predicted ratio $r(E,\theta_n)$ of the atmospheric $\nu_\mu$ and $\nu_e$ fluxes for i) the Earth core crossing and ii) only mantle crossing neutrinos, having trajectories for which $0.4 \lesssim \cos\theta_n \leq 1.0$, one has [21]: $r(E,\theta_z) \cong (2.0 - 2.5)$ for the neutrinos giving contribution to the sub-GeV samples of Super-Kamiokande events, and $r(E,\theta_n) \cong (2.6 - 4.5)$ for those giving the main contribution to the multi-GeV samples. If $s^2_{23} = 0.5$ and $r(E,\theta_z) \cong 2.0$, we have $(s^2_{23}\ r(E,\theta_z) - 1) \cong 0$, $((s^2_{23}\ r(E,\theta_z))^{-1} - 1) \cong 0$, and the possible effects of the $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_{\mu(\tau)}$ transitions on the $\nu_e$ and $\nu_\mu$ fluxes, and correspondingly in the sub-GeV $e-$like and $\mu-$like samples of events, would be strongly suppressed. The effects of interest are much larger for the multi-GeV neutrinos than for the sub-GeV neutrinos. They are also predicted to be larger for the flux of (and event rate due to) multi-GeV $\nu_e$ than for the flux of (and event rate due to) multi-GeV $\nu_\mu$.

The same conclusions are valid for the effects of oscillations on the fluxes of, and event rates due to, atmospheric antineutrinos $\bar{\nu}_e$ and $\bar{\nu}_\mu$.

Equations (3) - (4) and the similar equations for antineutrinos imply that in the case under study the effects of the $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, and $\nu_e \rightarrow \nu_{\mu(\tau)}$, $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu(\tau)}$, oscillations i) increase with the increase of $s^2_{23}$ and are maximal for the largest allowed value of $s^2_{23}$, ii) should be substantially larger in the multi-GeV samples of events than in the sub-GeV samples, and iii) in the case of the multi-GeV samples, for $\Delta m^2_{\rm atm} > 0$ ($\Delta m^2_{\rm atm} < 0$) they lead to an increase of the rate of $e^-$ ($e^+$) events and to a decrease of the $\mu^-$ ($\mu^+$) event rate. The last point follows from the fact that the magnitude of the effects we are interested in depends also on the 2-neutrino oscillation probabilities, $P_{2\nu}$ and $\bar{P}_{2\nu}$, and that $P_{2\nu}$ or $\bar{P}_{2\nu}$ (but not both probabilities) can be strongly enhanced by the Earth matter effects.

A more detailed analysis shows (see, e.g., refs. [13, 22]) that for $\Delta m^2_{\rm atm} = (2-3) \times 10^{-3}\ {\rm eV}^2 > 0$, the Earth matter effects can amplify $P_{2\nu}$ significantly when the neutrinos

cross *only the Earth mantle* i) for $E \sim (6-11)$ GeV, and ii) only for sufficiently long neutrino paths in the mantle, i.e., for $\cos\theta_n \gtrsim 0.4$. The magnitude of the matter effects increases with increasing of $\sin^2\theta_{13}$. The energy $E_{res}$ and the path length of neutrinos in the mantle, $L$, for which one can have $P_{2\nu} \cong 1$, are determined by the conditions:

$$E_{res} \cong 6.6 \left(\frac{\Delta m^2_{\rm atm}}{10^{-3}\ {\rm eV}^2}\right)\left(\frac{\rm N_A cm^{-3}}{\rm \bar{N}_e^{man}}\right)\cos 2\theta_{13}\ {\rm GeV}\ , \tag{6}$$

$$1.2\ \tan 2\theta_{13}\ \left(\frac{\rm \bar{N}_e^{man}}{\rm N_A cm^{-3}}\right)\left(\frac{L}{10^4\ {\rm km}}\right) = 1, \tag{7}$$

$\bar{N}_e^{man}$ and $N_A$ being the mean electron number density along the neutrino trajectory in the mantle and the Avogadro number.

In the case of atmospheric neutrinos crossing the Earth core, new resonance-like effects become apparent. For $\sin^2\theta_{13} < 0.05$ and $\Delta m^2_{\rm atm} > 0$, we can have $P_{2\nu} \cong 1$ *only due to the effect of maximal constructive interference between the amplitudes of the* $\nu_e \to \nu'_\tau$ *transitions in the Earth mantle and in the Earth core* [15, 23, 24]. The effect differs from the MSW one and the enhancement happens in the case of interest at a value of the energy between the resonance energies corresponding to the density in the mantle and that of the core [15]. The *mantle-core enhancement effect* is caused by the existence (for a given neutrino trajectory through the Earth core) of *points of resonance-like total neutrino conversion*, $P_{2\nu} = 1$, in the corresponding space of neutrino oscillation parameters [23, 24]. A rather complete set of values of $\Delta m^2_{\rm atm}/E$ and $\sin^2 2\theta_{13}$ for which $P_{2\nu} = 1$ for the Earth core-crossing atmospheric $\nu_\mu$ and $\nu_e$ was found in ref. [24]. The location of these points determines the regions where $P_{2\nu}$ is large, $P_{2\nu} \gtrsim 0.5$. For $\sin^2 2\theta_{13} < 0.10$, there is one set of values of $\Delta m^2_{\rm atm}/E$ and $\sin^2\theta_{13}$ for which $P_{2\nu} = 1$. This "solution" occurs for, e.g., $\theta_n = 0;\ 13^0; 23^0$, and $\Delta m^2_{\rm atm} = 2.0\ (3.0) \times 10^{-3}$ eV$^2$, at $\sin^2 2\theta_{13} = 0.034;\ 0.039;\ 0.051$, and $E \cong (2.8-3.1)$ GeV ($E \cong (4.2-4.7)$ GeV), see Table 2 in ref. [24].

# 3 Results

We use the method of calculation of the up-down (U-D) asymmetry in the Nadir angle ($\theta_n$) dependence of the ratio $N_\mu/N_e$ of the rates of the $\mu-$like and $e-$like multi-GeV events, $A(U-D)$, measured in water-Čerenkov detectors (Super-Kamiokande, Hyper-Kamiokande [25], etc.), described in ref. [13]. Our results are presented graphically in Fig. 1, where we show the asymmetry $A(U-D)$ as a function of $\sin^2 2\theta_{13}$, calculated for two intervals of values of $\cos\theta_n$, [0.40,084] ("mantle bin") and [0.84,1.0] ("core bin"), for $\Delta m^2_{\rm atm} = \pm 2 \times 10^{-3}$ eV$^2$ and for $\sin^2\theta_{23} = 0.36;\ 0.50;\ 0.64$. As Fig. 1 shows, for $\sin^2 2\theta_{13} \lesssim 0.06$, $A(U-D)$ in the *core bin* increases rapidly with $\sin^2 2\theta_{13}$, and remains practically constant for $0.06 \lesssim \sin^2 2\theta_{13} \leq 0.15$. For $\sin^2\theta_{23} = 0.64$ it reaches the values of (-0.45) for $\Delta m^2_{\rm atm} > 0$ (NH), and (-0.39) if $\Delta m^2_{\rm atm} < 0$ (IH) at $\sin^2 2\theta_{13} = 0.06$, while the asymmetry in the case of $2-\nu$ vacuum oscillations of the $\nu_\mu$ and $\bar{\nu}_\mu$ is considerably smaller in absolute value, (-0.28). In the case of $\sin^2\theta_{23} = 0.50$, the corresponding asymmetry values are (-0.42), (-0.38) and (-0.32), respectively. The asymmetry in the *mantle*

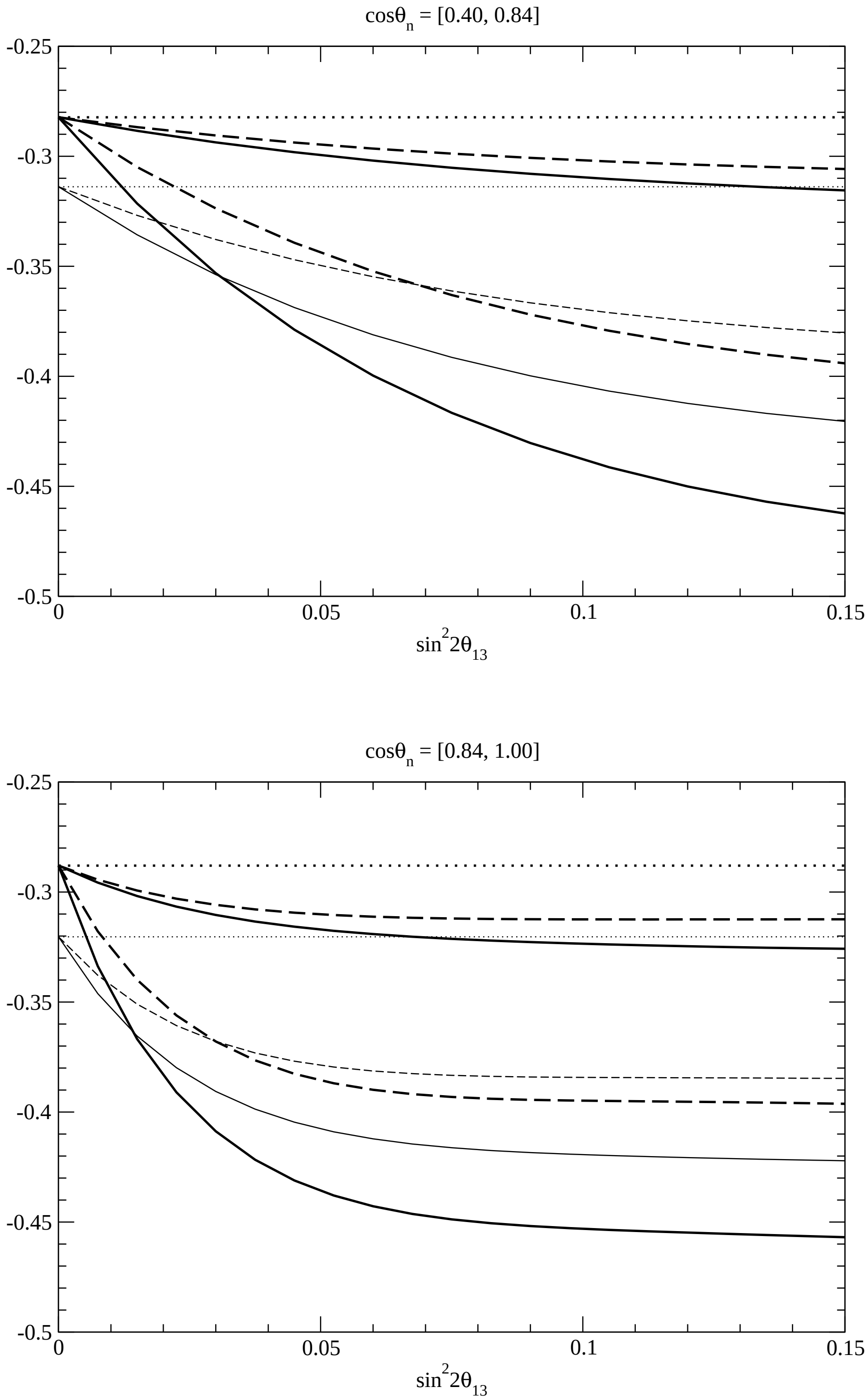


Figure 1: The up-down asymmetry in the Nadir angle dependence of the ratio $N_\mu/N_e$ of the rates of the $\mu-$like and $e-$like multi-GeV events measured in water-Čerenkov detectors, as a function of $\sin^2 2\theta_{13}$, calculated for two intervals of values of $\cos\theta_n$, [0.40,0.84] (*mantle bin* - upper panel) and [0.84,1.0] (*core bin* - lower panel), for $|\Delta m^2_{\rm atm}| = 2 \times 10^{-3}$ eV$^2$ and i) $\Delta m^2_{\rm atm} > 0$ - normal hierarchy (solid lines), ii) $\Delta m^2_{\rm atm} < 0$ - inverted hierarchy (dashed lines), and iii) 2-neutrino vacuum oscillations (dotted lines), and for $\sin^2\theta_{23}$ =0.36 (upper thick lines), 0.50 (thin lines), 0.64 (lower thick lines). In the case of vacuum oscillations, there is no distinction between $\sin^2\theta_{23}$ =0.36 and 0.64 (upper dotted line).

*bin* increases monotonically with the increase of $\sin^2 2\theta_{13}$. At $\sin^2 2\theta_{13} = 0.06$ it is smaller than the asymmetry in the *core bin*, but at $\sin^2 2\theta_{13} = 0.15$ the asymmetries in the *mantle* and the *core* bins practically coincide. For $\sin^2 \theta_{23} \sim 0.36$, the Earth matter effects in the subdominant neutrino oscillations are suppressed and the U-D asymmetries are essentially determined by their 2-neutrino vacuum oscillation values.

It is interesting to note that using the SK atmospheric neutrino data [8] one finds for the U-D asymmetry in the two mantle bins, $\cos\theta_n = [0.40, 0.60]$, $[0.60, 0.84]$ and in the core bin, respectively: $A_{m1}(U-D) = -0.29 \pm 0.13$, $A_{m2}(U-D) = -0.36 \pm 0.14$, $A_c(U-D) = -0.48 \pm 0.16$.

# 4 Conclusions

We have shown that the up-down asymmetry in the Nadir angle dependence of the ratio $N_\mu/N_e$ of the rates of $\mu-$like and $e-$like multi-GeV events measured in water-Čerenkov detectors (Super-Kamiokande, etc.) is sensitive to the Earth matter effects in the subdominant oscillations of the multi-GeV ($\sim (2-10)$ GeV) atmospheric neutrinos, $\nu_\mu \rightarrow \nu_e$, $\nu_e \rightarrow \nu_\mu$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$. The measurement with increased sensitivity of this asymmetry can provide fundamental information on the values of $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$, and on the sign of $\Delta m^2_{\rm atm}$ , i.e., on the neutrino mass hierarchy.

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